Name: ______________________________

Math 3053 Section 01 *Practice Final Exam* *November 19, 2019*

Follow the instructions for each question and show enough of your work so that I can follow your thought process. If I can't read your work, answer or there is no justification to a solution, you will receive little or no credit!

The items you must absolutely know for the exam:

(a) Showing sets are bijective, by construction or other means, to show a set is countable or uncountable

(b) What algebraic and transcendental numbers are

(c) What it means for numbers to be relatively prime

(d) What the gcd is of numbers and that it is an integer combination

(e) Fundamental Theorem of Arithmetic and Euclid's Lemma

1. It is known that the interval $(0, 1)$ is uncountable. Prove that the interval $(1, \infty)$ is uncountable by explicitly constructing a bijection from $(0, 1)$ to $(1, \infty)$. Be sure to prove the map you construct is really a bijection.

2. Show that $\mathbb{Z}$ is countable by constructing an explicit map from $\mathbb{N}$ to $\mathbb{Z}$ and showing it's a bijection.

3. Let $\alpha \in \mathbb{R}$. Let $p(x)$ be a polynomial with integer coefficients such that α is a transcendental number. Prove that $p(\alpha)$ must also be a transcendental number.

4. Let $\alpha \in \mathbb{R}$ be an algebraic number. Prove that α^{-1} can be expressed in terms solely consisting of sums with rational coefficients of powers of α.

5. Prove that $5n + 3$ and $7n + 4$ are relatively prime for all $n \in \mathbb{N}$.

6. Let m and n be integers that are relatively prime. Prove for any $r \in \mathbb{Z}$, there exists $x, y \in \mathbb{Z}$ such that $r = mx + ny$.

7. Let $d = \gcd(a, b)$ where $a, b \in \mathbb{N}$. If $a = da'$ and $b = db'$, show that $\gcd(a', b') = 1$.

8. Let $d = \gcd(a, b)$ where $a, b \in \mathbb{N}$. Prove that $\dfrac{a}{d}$ and $\dfrac{b}{d}$ are relatively prime.

9. We showed $\mathbb{R}$ is uncountable. Prove the interval $(0, 1)$ is uncountable by constucting a map from $(0, 1)$ to $\mathbb{R}$ and demonstrating the map is a bijection.